

On the Geometry Origin of Weak CP Phase

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Abstract

In this work, the postulation that weak CP phase originates in a certain geometry, is further discussed. Some intrinsic and strict constraints on the mixing angles, weak CP phase, and the Wolfenstein's parameters ρ and η are given by present data and the postulation itself. Especially, we predict $0.0076 \leq |V_{td}| \leq 0.0093$, $74.9^\circ \leq \gamma \leq 75.7^\circ$ when the corresponding inputs are at the 90% *C. L.*. The comparison of the predictions to the relevant experimental and theoretical results is listed. All the predictions coincide with the present experimental results and theoretical analysis very well.

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Quark mixing and CP violation [1, 2, 3, 4, 5, 6, 7] is one of the most interesting and important problem in particle physics. Up to now, the origin of CP violation is not clear to us. In the standard model, CP violation originates from a phase which presents in the CKM matrix [8, 9]. Mathematically, $\frac{(N-1)(N-2)}{2}$ phases are permitted to present in N by N unitarity matrix in addition to $\frac{N(N-1)}{2}$ Euler angles. However, physics is always in favour of more concise theory.

In the previous work [10], we have postulated that, the weak CP phase originates in a certain geometry. Here, we discuss further this issue. The central purpose is to make some predictions which can be tested by more precise data in B-factory in following few years.

To make this paper selfcontained, we begin with describing our postulation firstly.

1. The Postulation

A. CKM matrix in KM parametrization and $SO(3)$ rotation

There are many parametrization of the CKM matrix, such as the standard one advocated by the Particle Data Group [11, 12] and those given by Wolfenstein [13] etc. However, the original parametrization chosed by Kobayashi and Maskawa is more helpful to our understanding the problem, it is [9]

$$V_{KM} = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \quad (1)$$

with the standard notations $s_i = \sin \theta_i$ and $c_i = \cos \theta_i$. Note that, it is just the phase δ in V_{KM} violates CP symmetry. And all the three angles $\theta_1, \theta_2, \theta_3$ can be taken to lie in the first quadrant by adjusting quark field phases. In following discussions, we will fix the three angles in first quadrant.

It is easy to find that, the above matrix can be decomposed into a product of three Eulerian rotation matrice and one phase matrix [14].

$$V_{KM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & -s_2 \\ 0 & s_2 & c_2 \end{pmatrix} \begin{pmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -e^{i\delta} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{pmatrix}. \quad (2)$$

From the above equation, we can see that, the phase δ is inserted into the CKM matrix some artificially. Although it is permitted mathematically, it is not so natural physically.

Eq.(2) can easily remind us such a fact: the $SO(3)$ rotation of a vector. Let us describe this issue more detailed. We begin with a special example which has been written into many group theory textbooks. Suppose that vector \vec{V} locates on X -axis and parallel to Z -axis. Now, we move it to the Z -axis. There are infinite ways to do so. Here, as a special example, we consider two of the most special ways.

1. Rotate \vec{V} round Z -axis, after $\theta_1 = \pi/2$, it is rotated to Y -axis and still parallel to Z -axis. Then, continue to rotate it, but this time, the rotation is round X -axis. After $\theta_2 = \pi/2$, \vec{V} is moved to Z -axis, but now, it is anti-parallel to Y -axis. We denote it as \vec{V}_1 .

2. Rotate \vec{V} round Y -axis, after $\theta_3 = \pi/2$, it is moved to Z -axis directly, but it is anti-parallel to X -axis. We denote it as \vec{V}_2 .

Note that, all of the movements of the vectors described above are the parallel movements along the geodesics. Now, we find that, starting out from the same vector, through two different ways of rotation, we obtain two different vectors. The difference is only their direction. However, if we rotate \vec{V}_1 round Z -axis (for more general case, it is the normal direction of the point on which \vec{V}_1 and \vec{V}_2 stand), after $\delta = \pi/2$, we will get the same vector as \vec{V}_2 .

From the special example, it can be seen that, the result of twice non-coaxial rotations can not be achieved by one rotation. They are different from each other by a "phase" δ . For more general case, δ is given by a simple relation in spherical surface geometry

$$\sin \delta = \frac{(1 + \cos \theta_1 + \cos \theta_2 + \cos \theta_3) \sqrt{1 - \cos^2 \theta_1 - \cos^2 \theta_2 - \cos^2 \theta_3 + 2 \cos \theta_1 \cos \theta_2 \cos \theta_3}}{(1 + \cos \theta_1)(1 + \cos \theta_2)(1 + \cos \theta_3)}. \quad (3)$$

The geometry meaning of the above equation is shown in Fig.(1). δ is the solid angle enclosed by the three angles $\theta_1, \theta_2, \theta_3$ standing on a same point, or the area to which the solid angle corresponding on a unit spherical surface.

It is very important to note that, it has been realized that the magnitude of CP violation is closely related to a certain area more than ten years ago [5].

B. Phase, geometry and the weak CP phase

Due to Berry's famous work [15], the phase factor has aroused the theoretical physicists a great interests in the past fifteen years. People have realized that, the phase is closely related to a certain geometry or symmetry [16, 17]. For a non-trivial topology, the presence of the phase factor is natural. In quantum mechanics, The well known example is the Aharonov-Bohm effect [18].

To make the readers understand easily how we reached such a postulation - weak CP phase as a geometry phase, let us recall a simple fact in relativity [19][20].

Suppose that there are two observers A and B , A observes B , A gets the velocity \vec{V} of B , B observes A , B gets the velocity \vec{U} of A . It is evident that, $\vec{V} = -\vec{U}$, i.e. \vec{V} anti-parallel to \vec{U} . However, if the third observer C presents, and A and B observe each other not directly but through C , it will not be the above case. Suppose A observes B via C , that is, A solve the velocity of B by using the velocity of B relative to C and the one of C relative to A , A gets the velocity \vec{V}' of B . Similarly, B observes A via C , B gets the velocity \vec{U}' of A . Now, although $|\vec{V}'| = |\vec{U}'|$, $\vec{V}' \neq -\vec{U}'$, i.e., \vec{V}' and \vec{U}' are not parallel to each other. An angle presents between these two veclocity vectors. This matter is illustrated in Fig.(2).

What can we learn from the above example?

First, the presence of the angle is closely related to the presence of the third observer. This is very similar to the case of quark mixing. If we only have two generations of quark, we have no the weak CP phase, but, once we have three generations of quark, the weak CP phase will

present. It is just this point stimulates us relating the weak CP phase to the geometry phase.

Second, the more important issue we should realize here is that, although the space in which the three observers exist is flat, the velocity space is hyperboloidal. Or in other words, it is a non-trivial topology. In such geometric spaces, the presence of the phase is very natural [16, 20, 19, 21, 22, 23].

Now, if we notice that mathematically, $SO(3) \simeq S^2$ or $SU(2)/U(1) \simeq SO(3) \simeq S^2$, and if the quarks have a certain kind of $SO(3)$ hidden symmetry, then the phase can present, and the spherical geometry relation Eq.(3) can be obtained.

C. The postulation in standard parametrization

As described above, we postulate an ad hoc relation. It is, the three mixing angles $\theta_1, \theta_2, \theta_3$ and the weak CP phase δ satisfy Eq.(3).

If we use the standard parametrization [11, 12] instead of KM parametrization Eq.(1), and correspondingly, we transform the constraint Eq.(3) into the one expressed by $\delta_{13}, \theta_{12}, \theta_{23}$ and θ_{13} , then it will be more convenient and clear for the following discussions.

The standard parametrization is

$$V_{KM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \quad (4)$$

with $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$ for the "generation" labels $i, j = 1, 2, 3$. As the KM parametrization, the real angles θ_{12}, θ_{23} and θ_{13} can all be made to lie in the first quadrant. The phase δ_{13} lies in the range $0 < \delta_{13} < 2\pi$. In following, we will also fix the three angles θ_{12}, θ_{23} and θ_{13} in first quadrant.

The corresponding constraint on $\delta_{13}, \theta_{12}, \theta_{23}$ and θ_{13} - or, the expression of our postulation in this parametrization is

$$\sin \delta_{13} = \frac{(1 + s_{12} + s_{23} + s_{13})\sqrt{1 - s_{12}^2 - s_{23}^2 - s_{13}^2 + 2s_{12}s_{23}s_{13}}}{(1 + s_{12})(1 + s_{23})(1 + s_{13})} \quad (5)$$

From the transformation relation between the KM parametrization and the standard one, we immediately find the following symmetry between these two parametrization

$$c_1 \rightleftharpoons s_{13} \quad s_1 \rightleftharpoons c_{13} \quad c_2 \rightleftharpoons s_{23} \quad s_2 \rightleftharpoons c_{23} \quad c_3 \rightleftharpoons s_{12} \quad s_3 \rightleftharpoons c_{12} \quad \delta \rightleftharpoons \delta_{13}$$

$$\text{KM parametrization} \rightleftharpoons \text{Standard parametrization}$$

Then, Eq.(5) can be got by simple substitutions.

2. Further investigation

According to our postulation, only three elements in the CKM matrix are independent. So, if we can determine experimentally three elements of the CKM matrix, we should then determine

the whole CKM matrix. This assertion can be checked by reproducing the whole matrix in certain error ranges with only three of the elements as inputs.

The programme is:

- a. For each group of given V_{us} , V_{ub} and V_{cb} , solve s_{12} , s_{23} , s_{13} from the following equations

$$|V_{us}| = s_{12} c_{13} \quad |V_{ub}| = s_{13} \quad |V_{cb}| = s_{23} c_{13}. \quad (6)$$

- b. Substituting Eq.(5) into CKM matrix Eq.(4). Then, solve the moduli of all the elements.

- c. Let V_{us} , V_{ub} and V_{cb} vary in certain ranges. Repeat the steps a and b.

When we let V_{us} , V_{ub} and V_{cb} vary in the ranges [11]

$$0.217 \leq V_{us} \leq 0.224 \quad 0.0018 \leq V_{ub} \leq 0.0045 \quad 0.036 \leq V_{cb} \leq 0.042 \quad (7)$$

we get all the magnitudes of the elements as

$$\begin{pmatrix} 0.9746 \sim 0.9762 & \underline{0.217 \sim 0.224} & \underline{0.0018 \sim 0.0045} \\ 0.2168 \sim 0.2239 & 0.9737 \sim 0.9755 & \underline{0.036 \sim 0.042} \\ 0.0076 \sim 0.0093 & 0.0352 \sim 0.0413 & 0.9991 \sim 0.9994 \end{pmatrix} \quad (8)$$

Compare with those given by [11]

$$\begin{pmatrix} 0.9745 \sim 0.9760 & 0.217 \sim 0.224 & 0.0018 \sim 0.0045 \\ 0.217 \sim 0.224 & 0.9737 \sim 0.9753 & 0.036 \sim 0.042 \\ 0.004 \sim 0.013 & 0.035 \sim 0.042 & 0.9991 \sim 0.9993 \end{pmatrix} \quad (9)$$

we find that, the predicted results are well in agreement with those given by data book. We have reproduced the whole CKM matrix succesfully with only three of its elements as inputs.

In the meantime, $|V_{td}|$ is not sensitive to the variations of the inputs. It lies in a very narrow window with the central value being about ~ 0.0085 , even if we take a little more large error ranges for the inputs. This may be taken as one of the criterions to judge our postulation.

On the other hand, the relevant result extracted from the experiment on $B_d^0 - \overline{B}_d^0$ mixing gives [11]

$$|V_{tb}^* \cdot V_{td}| = 0.0084 \pm 0.0018. \quad (10)$$

we find that, the prediction about $|V_{td}|$ based on our postulation, not only coincide with the experimental result very well, but also gives a strict constraint.

3. Predictions based on the postulation

What can we extract from this postulation? And, how about the correctness of the conclusions extracted from the postulation? Let us list some other simple conclusions for more strict tests in future.

A. On the three mixing angles in CKM matrix

To make θ_1, θ_2 and θ_3 ($0 < \theta_i < \pi/2$, $i = 1, 2, 3$) enclose a solid angle, the following relation among them should be satisfied.

$$\theta_i + \theta_j \geq \theta_k \quad (i \neq j \neq k \neq i = 1, 2, 3) \quad (11)$$

Comparing Eq.(5) with Eq.(3), we find that, δ_{13} is the solid angle enclosed by $(\pi/2 - \theta_{12})$, $(\pi/2 - \theta_{23})$ and $(\pi/2 - \theta_{13})$. Hence, for the standard parametrization, the following relation should hold

$$(\frac{\pi}{2} - \theta_{ij}) + (\frac{\pi}{2} - \theta_{jk}) \geq (\frac{\pi}{2} - \theta_{ki}) \quad (i \neq j \neq k \neq i = 1, 2, 3. \quad \theta_{ij} = \theta_{ji}). \quad (12)$$

It can be checked that, Eqs.(11, 12) are easily satisfied by the present experimental data [11].

B. On the weak phase δ_{13}

According to the geometry meaning of δ (δ_{13}), it is the solid angle enclosed by $\theta_1, \theta_2, \theta_3$ ($\frac{\pi}{2} - \theta_{12}, \frac{\pi}{2} - \theta_{23}, \frac{\pi}{2} - \theta_{13}$). So, if $0 < \theta_i < \pi/2$ ($0 < \theta_{ij} < \pi/2$ $i \neq j = 1, 2, 3$) and $\theta_i + \theta_j > \theta_k$ ($(\frac{\pi}{2} - \theta_{ij}) + (\frac{\pi}{2} - \theta_{jk}) \geq (\frac{\pi}{2} - \theta_{ki})$ $i \neq j \neq k \neq i = 1, 2, 3$ $\theta_{ij} = \theta_{ji}$) are satisfied, then, δ (δ_{13}) only can lie in the first quadrant. At most, one can take the solid angle δ (δ_{13}) as $4\pi - \delta$ ($4\pi - \delta_{13}$). So, $2\pi - \delta$ ($2\pi - \delta_{13}$) in the fourth quadrant can also be permitted. Hence,

The second and third quadrants for δ (δ_{13}) are excluded thoroughly.

The recent analysis of Buras, Jamin and Lautenbacher [24] indicates that, $\sin \delta_{13}$ likely lies in the first quadrant.

C. On the three angles in unitarity triangle

The three angles α , β and γ in the unitarity triangle defined as [7]

$$\alpha \equiv \arg(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}) \quad \beta \equiv \arg(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}) \quad \gamma \equiv \arg(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}) \quad (13)$$

If a small change being made, it is easy to see that, the defined angles are composed of squared and quartic invariants. For example,

$$\alpha \equiv \arg(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}) = \arg(-\frac{V_{td}V_{ud}^*V_{tb}^*V_{ub}}{V_{ud}V_{ud}^*V_{ub}V_{ub}^*})$$

the numerator in the definition is a quartic invariant and the denominator is a product of two squared invariants.

Firstly, we can directly estimate that angle γ is about $\pi/2$. According to the geometric meaning of δ_{13} , it is the solid angle enclosed by $(\pi/2 - \theta_{12})$, $(\pi/2 - \theta_{23})$ and $(\pi/2 - \theta_{13})$. The up-to-date experimental data [11] tells us that, $s_{12} = 0.217$ to 0.222 , $s_{23} = 0.036$ to 0.042 , and $s_{13} = 0.0018$ to 0.0014 . It means that, θ_{12}, θ_{23} and θ_{13} are very small. Approximate to the first order, we can take δ_{13} as the solid angle enclosed by three right angles. So we get $\delta_{13} \sim \pi/2$. On the other hand, based on the definition of γ in Eq.(13) and the form of standard parametrization Eq.(4), it is evident that, $\gamma \sim \delta_{13}$. Finally, with no any detailed calculation, we have got to know that, $\gamma \sim \pi/2$ when approximate to the first order.

Now, let us investigate the three angles carefully. The programme is similar to that in section 2.

- a. For each group of given V_{us} , V_{ub} and V_{cb} , solve s_{12} , s_{23} , s_{13} from Eq.(6).
- b. Substituting Eq.(5) into CKM matrix Eq.(4). Then, solve all of the elements with the results of a being used.
- c. Solve α , β and γ according to the definition Eq.(13).
- d. Let V_{us} , V_{ub} and V_{cb} vary in certain ranges. Repeat the steps a , b and c .

We still let V_{us} , V_{ub} and V_{cb} vary in the ranges given by Eq.(7). The corresponding outputs are

$$72.1^0 \leq \alpha \leq 94.2^0 \quad 10.7^0 \leq \beta \leq 32.4^0 \quad 74.9^0 \leq \gamma \leq 75.7^0. \quad (14)$$

The recent analysis with more information such as a fit of $B_d - \bar{B}_d$ mixing and ϵ being considered by Buras gives [26]

$$35^0 \leq \alpha \leq 115^0 \quad 11^0 \leq \beta \leq 27^0 \quad 41^0 \leq \gamma \leq 134^0 \quad (15)$$

or more strictly

$$70^0 \leq \alpha \leq 93^0 \quad 19^0 \leq \beta \leq 22^0 \quad 65^0 \leq \gamma \leq 90^0. \quad (16)$$

It is easy to find, similar to V_{td} in section 2, we obtain a more strict constraint on γ . We predict a very narrow window for γ with the central value about $\sim 75.3^0$. Furthermore, all the predictions about α , β and γ coincide with the relevant analysis [7, 25, 26].

D. On the phases in the case of more than three generations

For the case of more than three generations, the number of the independent phases is also $(n-1)(n-2)/2$, where n is the number of the generations. According to the geometry meaning of the phase, the number of the independent phase is equal to the number of the triangles which we can draw among n points on a spherical surface with the areas of the triangles are independent.

We consider the case of the four generations as a special example, the geometry picture is shown in Fig.(3). In fact, we can draw four spheric triangles $\triangle ABC, \triangle BCD, \triangle CDA, \triangle DAB$ among the four vertexes A, B, C and D . But, due to the constraint $S_{\triangle ABC} + S_{\triangle ADC} = S_{\triangle BAD} + S_{\triangle BCD}$, only three of them are independent. In the meantime, the constraints $S_{\triangle ABC} + S_{\triangle ADC} = S_{\triangle BAD} + S_{\triangle BCD}$ and $\theta_i + \theta_j > \theta_k (i \neq j \neq k = 1, 2, \dots, 6, 0 < \theta_i < \pi/2$ and $\theta_i, \theta_j, \theta_k$ can enclose a solid angle) give limits on the six Euler angles in the CKM matrix.

Starting out from the geometry picture, we can extract some useful informations about the mixing angles between the fourth and the first three generations based on the mixing angles among the first three generations. For instance, we have

$$\theta_{14} + \theta_{24} + \theta_{34} < \frac{3}{4}\pi + \frac{1}{2}(\theta_{12} + \theta_{23} + \theta_{13})$$

where, the definitions of θ_{14} , θ_{24} , θ_{34} are similar to θ_{12} , θ_{23} , θ_{13} in Eq.(4). Substituting the present data [11], we obtain $\theta_{14} + \theta_{24} + \theta_{34} < 142.7^\circ$.

E. On the Wolfenstein's parameters η and ρ

To make it be convenient to use the CKM matrix in the concrete calculations, Wolfenstein parametrized it as [13]

$$V_W = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta + i\eta\frac{1}{2}\lambda^2) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 - i\eta A^2\lambda^4 & A\lambda^2(1 + i\eta\lambda^2) \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}. \quad (17)$$

Actually, one can take different parametrization in different cases. They are only for the convenience in discussing the different questions, but the physics does not change when adopting various parametrizations.

According to Buras etc., there is a very nice corresponding relation between Wolfenstein's parameters and the ones in the standard parametrization. It reads [27]

$$s_{12} = \lambda, \quad s_{23} = A\lambda^2, \quad s_{13}e^{-i\delta_{13}} = A\lambda^3(\rho - i\eta). \quad (18)$$

So,

$$s_{13} = A\lambda^3\sqrt{\rho^2 + \eta^2}, \quad \sin \delta_{13} = \frac{\eta}{\sqrt{\rho^2 + \eta^2}} \quad (19)$$

and consequently,

$$\rho = \frac{s_{13}}{s_{12}s_{23}} \cos \delta_{13}, \quad \eta = \frac{s_{13}}{s_{12}s_{23}} \sin \delta_{13}. \quad (20)$$

In Eq.(18), λ and A are the two better known parameters. But, because of the uncertainty of hadronic matrix elements and other reasons, it is difficult to extract more information about ρ and η from experimental results. Up to now, we still know little about them. More than ten years ago, Wolfenstein estimated that the upper limit on η is about 0.1 [13], but the recent analysis indicate that, ρ and η are about [28, 29, 30]

$$-0.15 < \rho < 0.35, \quad 0.20 < \eta < 0.45. \quad (21)$$

Now that the four angles in CKM matrix are not independent, the four Wolfenstein's parameters A , λ , ρ and η must also be not independent.

Substituting Eqs.(18-19) into Eq.(5), it is easy to achieve

$$\frac{\eta}{\sqrt{\rho^2 + \eta^2}} = 1 - \frac{\lambda^2}{2} - A\lambda^3 + \lambda^4(-\frac{1}{8} + A - \frac{A^2}{2} - A\sqrt{\rho^2 + \eta^2}). \quad (22)$$

This is just the geometry constraint on Wolfenstein's parameters when approximate to the fourth order of λ .

In following, let us investigate carefully the permitted ranges of ρ and η by present data. If we start out from Eq.(22) directly, and take [11, 31]

$$\lambda = 0.2196 \pm 0.0023 \quad A = 0.819 \pm 0.035$$

as inputs, then we can obtain the dependence of η on ρ . The result is shown in Fig.(4). It can be seen from the figure that, η and ρ satisfy an approximate linear relation.

We can also begin with Eq.(20). But, we should know the three mixing angles firstly. This can be achieved by use of three of the CKM matrix elements such as V_{us} , V_{ub} and V_{cb} . In section 2, we have found that, the whole matrix can be reconstructed very well based only on three of the elements and Eq.(5). Once the three mixing angles are determined, we can extract the dependence of η on ρ again from Eq.(20). We take the relevant inputs from the data book [11] as Eq.(7). The numerical result is also shown in Fig.(4). We find it is just a little part of that drawn from Eq.(22).

Now, we can read from the figure that, when all the three inputs V_{us} , V_{ub} and V_{cb} are taken at 90% *C. L.*, we obtain the outputs

$$0.048 < \rho < 0.140, \quad 0.18 < \eta < 0.54. \quad (23)$$

Comparing with Eq.(21), the range for ρ is more narrow while the range for η is relative wide. However, with more precise measurement on the relevant CKM matrix elements in future, we can determine them more accurately.

4. Conclusions and discussions

Based on the postulation that weak CP phase originates in a certain geometry, some intuitive results are obtained. We summarize the results as following.

1. There exists a intrinsic constraint on the three mixing angles in CKM matrix. It is

$$\theta_i + \theta_j \geq \theta_k \quad (i \neq j \neq k \neq i = 1, 2, 3)$$

or

$$\left(\frac{\pi}{2} - \theta_{ij}\right) + \left(\frac{\pi}{2} - \theta_{jk}\right) \geq \left(\frac{\pi}{2} - \theta_{ki}\right) \quad (i \neq j \neq k \neq i = 1, 2, 3. \quad \theta_{ij} = \theta_{ji})$$

2. Predict undoubtedly that, if all the mixing angles are made to lie in the first quadrant, the second and third quadrant for δ (δ_{13}) are excluded thoroughly.

3. The 90% *C.L.* inputs V_{us} , V_{ub} and V_{cb} gives

$$72.1^0 \leq \alpha \leq 94.2^0 \quad 10.7^0 \leq \beta \leq 32.4^0 \quad 74.9^0 \leq \gamma \leq 75.7^0$$

for the unitarity triangle.

4. For the case of more than three generations, the numbers of the independent phases is equal to that given by Kobayashi-Maskawa theory. If the fourth generation exists, the present data gives a limit on the mixing angles between the fourth and the first three generations as

$$\theta_{14} + \theta_{24} + \theta_{34} < 142.7^0$$

5. The constraint on the Wolfenstein's parametrization is worked out, and the present data implies that

$$0.048 < \rho < 0.140, \quad 0.18 < \eta < 0.54.$$

We find that all the predictions coincide with the present data and the relevant analysis very well.

Especially, the postulation gives strict constraints on the moduli of CKM matrix element $|V_{td}|$ and the angle γ in unitarity triangle. They are predicted as

$$0.0076 \leq |V_{td}| \leq 0.0093, \quad 74.9^\circ \leq \gamma \leq 75.7^\circ.$$

These results can be taken as the key criterions to judge our postulation in next two years.

Our postulation can be further put to the more precise tests in B -factory in following few years. If it can be verified finally, it means that, only three elements in CKM matrix are independent, and hence we can remove one of the free parameters in the standard model. If then, we will feel that the physics is more simple, natural, and beautiful. Besides, it can provide us some hints to the hidden symmetry. But, we will naturally ask, what is the dynamic origin?

Although our postulation is supported by the present experimental results. It is a ad hoc supposition now, it still needs the further verification by experiments and the basic theory on which it can base. The further theoretical work on this problem is under way.

Because the CP phase can originate from many ways in different theories and physical processes, we hope that our postulation at least provide partial origin of the CP violation even if it is not the whole origin. At least, it is a good parametrization for the weak CP phase.

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Figure 1: The solid angle and the spheric triangles. Where, $\alpha(= AOB)$, $\beta(= BOC)$, $\gamma(= COA)$ represent θ_1 , θ_2 , θ_3 . δ is the area of the spheric triangle $\triangle ABC$, or the solid angle enclosed by the three angles AOB , BOC , and COA .

Figure 2: If there are only two observers A and B , then, $\vec{U} = -\vec{V}$. When the third observer C presents, and we know that the velocities of C relative to A and the one of B relative to C are \vec{V}_{AC} and \vec{V}_{CB} respectively, the velocities of C relative to B and the one of A relative to C are $\vec{V}_{BC}(= -\vec{V}_{CB})$ and $\vec{V}_{CA}(= -\vec{V}_{AC})$ respectively, then we can solve the velocities of B relative to A and the one of A relative to B , which are denoted by \vec{V}' and \vec{U}' respectively. Now, we find that $\vec{V}' \neq -\vec{U}'$, an angle δ presents between them.

Figure 3: The geometry picture for the case of the four generations. Where, θ_i ($i = 1, 2, \dots 6$.) are the angles AOB , AOC , AOD , BOC , BOD and COD . δs are the areas of the spheric triangles enclosed by any three of the vertexes A , B , C and D , or the solid angles enclosed by the angles such as AOB , AOC , BOC etc.. Because $S_{ABC} + S_{CDA} = S_{BCD} + S_{DAB}$, only three of the four solid angles (or the areas of the spheric triangles) are independent. Here, S_{IJK} represents the area of the spheric triangle enclosed by the vertexes I , J and K .

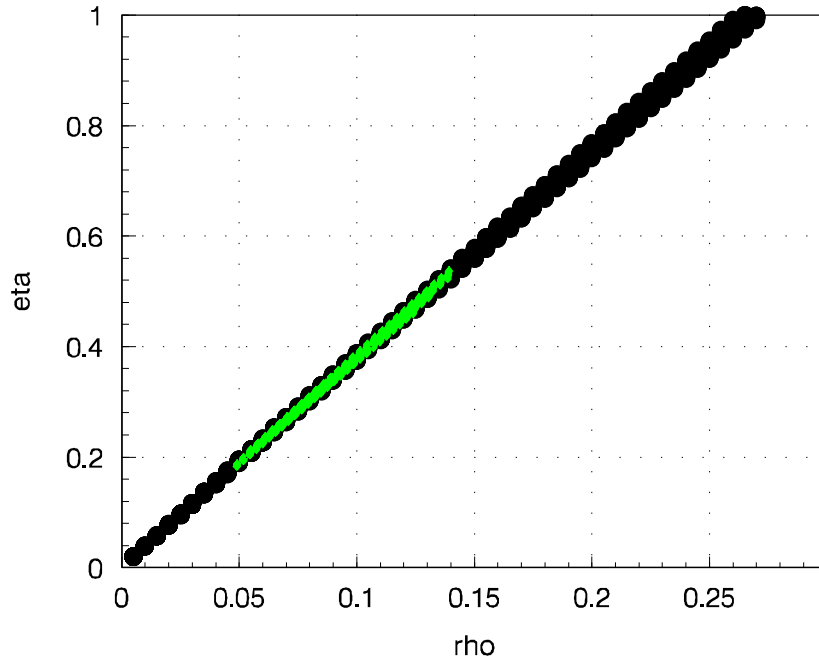


Figure 4: The dependence of η on ρ based on Eq.(22) and the permitted ranges for them by the present data. Here, the errors of the inputs have been considered.